1. INTRODUCTION

It has been reported by Pugh[1] that some metals, notably zinc and bismuth, which are normally brittle in tension at NTP, become markedly ductile when tested under hydrostatic pressure. This transition from brittle to ductile behaviour is very sharp, the specimens being either very brittle or very ductile. Galli and Gibb[2] have suggested the empirical relationship between the brittle to ductile transition temperature, T_c, and hydrostatic pressure, p, for molybdenum

$$T_c = \frac{A}{\ln (\sigma_f + p) + B}, \tag{1}$$

where A and B are constants and σ_f is the fracture stress at atmospheric pressure. Pugh[3] suggested for zinc the empirical relationship

$$T_c = A' - B'p, \tag{2}$$

where A' and B' are constants. A theory for the transition is given below.

2. THEORY OF THE TRANSITION PRESSURE

Near the brittle to ductile transition, brittle fracture occurs when the applied stress is sufficiently high to propagate a dislocation crack in a Griffith manner. If the applied stress is not high enough for this, fracture may occur by the linkage of many small cracks. It is thought that the propagation of a crack requires more energy than its initiation[4,5], and the analysis is based on this criterion. The calculation is similar to one due to Petch for the brittle-ductile transition temperature in mild steel[6].

Consider a crack of length l at an angle θ to the slip plane, wedged open by n dislocations of Burger's vector, b, under the action of the applied tensile stress, σ, at an angle $\pi/4$ to the slip plane, Fig. 1 (the most favoured slip lines are those at $\pi/4$ to the tensile stress). Let the applied hydrostatic pressure be p. If we assume that the hydrostatic pressure affects only the work done in opening the crack, then the energy, W, of the crack under this stress system, represented in Fig. 1, is composed of the following.

(1) The elastic energy of the stress field set up by the crack; this has been calculated by Stroh[7] to be $[(n^2b^2G)/\{4\pi(1-\nu)\}] \ln 4r/l$, where G is the rigidity modulus, ν the Poisson's ratio and r the effective radius of the stress field.

(2) The surface energy of the crack; this is given by $2l\gamma'$, where γ' is the effective surface energy of the crack. This effective surface energy term includes the plastic work associated with the growth of the crack, and may be much greater than the true surface energy.

(3) The elastic energy of the crack in the applied stress field; Stroh[8] calculates this to be

$$-\frac{\pi(1-\nu)\sigma^2 l^2}{8G}.$$

The work of Sack[9] suggests that no great error results in ignoring the effect of the hydrostatic pressure.

(4) The energy due to the increase in volume on opening the crack; this

consists of two parts, namely (a) the work done by the component of tensile stress normal to the crack, $(-nbl/2)\sigma \sin\{\theta - (\pi/4)\}$, and (b) the work done against the hydrostatic pressure $(nbl/2)p$.

The total energy of the crack is thus

$$W = \frac{n^2b^2G}{4\pi(1-\nu)} \ln \frac{4r}{l} + 2l\gamma' - \frac{\pi(1-\nu)\sigma^2 l^2}{8G} - \frac{nbl}{2}\sigma \sin\left(\theta - \frac{\pi}{4}\right) + \frac{nbl}{2}p. \quad (3)$$

Strictly, this equation applies only to a two-dimensional model but Sack[9], who extended the argument to a penny-shaped crack, showed that the difference will be a numerical factor only.

For the crack to spread under the applied stress, W must decrease as l increases, and the length of the crack at equilibrium will be given by $dW/dl = 0$. Thus

$$\frac{dW}{dl} = 0 = -\frac{n^2b^2G}{4\pi(1-\nu)}\frac{1}{l} + \frac{nb}{2}\left\{\frac{4\gamma'}{nb} + p - \sigma \sin\left(\theta - \frac{\pi}{4}\right)\right\} - \frac{\pi c}{4G}(1-\nu)\sigma^2. \quad (4)$$

Rearranging equation (4) we get

$$\frac{\pi}{4G}(1-\nu)\sigma^2 l^2 - \frac{nb}{2}\left\{\frac{4\gamma'}{nb} + p - \sigma \sin\left(\theta - \frac{\pi}{4}\right)\right\} l + \frac{n^2b^2G}{4\pi(1-\nu)} = 0. \quad (5)$$

The critical length of crack will occur when the roots of equation (5) are equal, and this will happen when

$$\frac{4\gamma'}{nb} + p_c - \sigma \sin\left(\theta - \frac{\pi}{4}\right) = \sigma, \quad (6)$$

where p_c is now the critical hydrostatic pressure for the transition from brittle to ductile fracture. Rearranging equation (6) we get as the critical condition

$$nb\left\{\sigma + \sigma \sin\left(\theta - \frac{\pi}{4}\right) - p_c\right\} = 4\gamma'. \quad (7)$$

If the left-hand side of equation (7) exceeds $4\gamma'$, then the crack will spread catastrophically. In practice, especially with the more ductile metals, plastic blunting of the crack may occur, in which case a higher value of σ would be required for a given p.

The value of n is obtained from the work of Eshelby, Frank and Nabarro[10]. If a dislocation source is activated near the centre of a grain of diameter d, the maximum length of slip plane on which a crack can form is $d/2$. The applied shear stress will be $\frac{1}{2}(\sigma - \sigma_0)$, where $\frac{1}{2}\sigma_0$ is the frictional shear stress opposing the motion of a free dislocation. The number of dislocations, n', that can be packed into a length $d/2$ under this applied stress is given by

$$\frac{1}{2}(\sigma - \sigma_0) = \frac{2Gn'b}{\pi(1-\nu)d}.$$

or

$$n' = \frac{\pi}{4Gb}(1-\nu)(\sigma - \sigma_0)d. \quad (8)$$

Now Stroh[8] has shown that the most difficult step in the coalescence of dislocations is the coalescence of the first two, the stress to add subsequent dislocations falling progressibely. Therefore very few dislocations will remain